Effect of surfactant on the long-wave instability of a shear-imposed liquid flow down an inclined plane

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The effect of an insoluble surfactant on the linear stability of a shear-imposed flow down an inclined plane is examined in the long-wavelength limit. It has been known that a free falling film flow with surfactant is stable to long-wavelength disturbances at sufficiently small Reynolds numbers. Imposing an additional interfacial shear, however, could cause instability due to the shear-induced Marangoni effect. Two modes of the stability are identified and the corresponding growth rates are derived. The underlying mechanisms of the stability are also elucidated in detail. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1823171]

I. INTRODUCTION

The hydrodynamic stability of a single liquid layer has been of interest in many engineering applications such as those occurring in coating processes[1] and pulmonary fluid mechanics.[2-5] Surface-active agents often play critical roles in these applications, affecting the stability of liquid layer.

In the absence of surfactant, Yih[6] clearly demonstrated that an inclined liquid layer was stable to long-wavelength disturbances at sufficiently small Reynolds numbers. For a surfactant-covered liquid flow down an inclined plane, there are only a few studies examining the interplay between Marangoni effects and gravity-driven base flows. Whitaker and Jones[7] and Lin[8] employed the long-wavelength analysis and demonstrated that surfactant could have a stabilizing effect, since the critical Reynolds number increased with surfactant elasticity. Pozrikidis[9] recently examined the same system for arbitrary wavelengths of disturbances in the limit of Stokes flow. He showed that although the system with surfactant still remained stable, it was less stable compared to the surfactant-free case. Ji and Setterwall[10] numerically studied the impact of soluble surfactants on the linear stability of a vertically falling film. They found that Marangoni effects destabilized the system for moderate or short waves in the low-Reynolds-number regime.

The interaction between surfactant and base flow appears in the stability analysis through the interfacial tangential stress condition and the surfactant transport equation. The first is due to a jump in the basic shear stress across the interface while the second is primarily reflected by a perturbation of the basic interfacial velocity. A gravity-driven base flow has no impact from the second mechanism because its zero interfacial stress leads to a vanishing perturbation of the basic interfacial velocity.

The surfactant-induced Marangoni instability solely due to base flows with *nonzero* interfacial stresses has been recently demonstrated in the studies by Frenkel and Halpern[11,12] for the linear stability of a surfactant-laden, two-layer Poiseuille–Couette flow in the limit of Stokes flow. They discovered that the presence of surfactant could induce destabilization to the system that is otherwise stable in the

absence of surfactant. The nonlinear stability analysis performed by Blyth and Pozrikidis[13] later demonstrated that such Marangoni-induced instability can be arrested by nonlinear effects. It was suggested in Ref. 12 that the reason why the previous studies of inclined flow with surfactant[7,8] did not indicate instability is due to the fact that the basic interfacial shear is zero. When such a system is subject to an additional interfacial stress, the instability seems to hinge on how an imposed shear acts and modifies the way that flows interact with surfactant. This is the central issue of the present study.

The motivation of the current study arises from efforts to construct an appropriate model to understand the dynamics of a surfactant-laden lining liquid flow in an airway as occurring in airway occlusion processes[2,3] or bolus-dispersal surfactant replacement therapy.[4,5] When the effect of gravity is negligible, although a shear flow can induce the Marangoni destabilization, its exerting directions are clearly irrelevant to the stability. For the situation in large airways, however, a liquid layer flow is not only driven by gravity, but also could undergo an interfacial stress introduced by an airflow that acts in a direction either along or opposing to gravity during breathing.[14] The instability seems to depend on how a surfactant interacts with such base flows under various flow-driven conditions. In this paper we shall *ab initio* address this issue using the long-wavelength stability analysis.

II. MATHEMATICAL FORMULATION

Consider a surfactant-laden, incompressible liquid layer with density ρ and viscosity μ flowing down a plane with an inclined angle θ_0. An additional constant shear stress τ_s^* induced by an airflow is exerted on the air-liquid interface and its direction can either assist or oppose the gravity-driven flow. The base state configuration consists of a liquid layer with a uniform thickness of h and an air-liquid interface coated by an insoluble surfactant with a uniform concentration Γ_0^*. We define x^* to be the coordinate along the plane and y^* to be the coordinate perpendicular to the liquid layer with $y^* = 0$ defining the plane, as shown in Fig. 1. Let u^* and
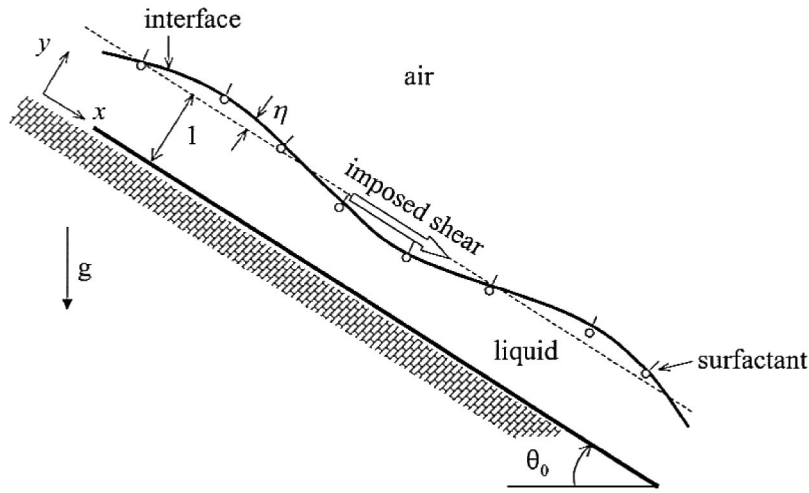
FIG. 1. Geometry of a surfactant-laden, shear-imposed liquid flow down an inclined plane.

$v*$ denote the velocity components in the $x*$ and $y*$ directions, respectively, and let $p*$ be the pressure. The base flow is given by

$$\bar{u}* = \frac{\rho g \sin \theta_0 h^2}{2\mu}\left[2\left(\frac{y*}{h}\right) - \left(\frac{y*}{h}\right)^2\right] + \frac{\tau_s^*}{\mu}y*. \tag{1}$$

We choose h as the characteristic length and scale the velocities with respect to the basic interfacial velocity U_s^* $=\rho g h^2 \sin \theta_0/2\mu$. The time scale is h/U_s^* and the pressure is scaled by $\rho g h \sin \theta_0/2$. The surfactant concentration has a scale of Γ_0^*. Then the base states in the dimensionless form become

$$\bar{u} = (2y - y^2) + \tau_s y, \quad \bar{p} = p_0 + 2\cot\theta_0(1 - y), \quad \bar{\Gamma} = 1, \tag{2}$$

where $\tau_s = \tau_s^* h/\mu U_s^*$ is the dimensionless imposed interfacial stress and p_0 is a constant pressure of air. Let σ denote the dimensionless surface tension scaled by the surface tension σ_0^* corresponding to the base-state surfactant concentration Γ_0^*. The dimensionless equation of state is assumed to have the form

$$\sigma = 1 - E(\Gamma - 1), \tag{3}$$

where $E = -\Gamma_0^*/\sigma_0^*(\partial\sigma*/\partial\Gamma*)_{\Gamma_0}$ is the dimensionless surfactant elasticity.

Since the linear stability analysis can be considered only two dimensional by appealing to Squire's theorem,[12] we start with the complete governing equations and boundary conditions for a two-dimensional system. The dimensionless governing equations for the fluid motion are the continuity equation and the Navier–Stokes equations. They are given by

$$u_x + v_y = 0, \tag{4}$$

$$\mathrm{Re}(u_t + uu_x + vu_y) = -p_x + 2 + u_{xx} + u_{yy}, \tag{5}$$

$$\mathrm{Re}(v_t + uv_x + vv_y) = -p_y - 2\cot\theta_0 + v_{xx} + v_{yy}, \tag{6}$$

where $\mathrm{Re} = \rho U_s^* h/\mu$ is the Reynolds number. The system is subject to the following boundary conditions. The velocity vanishes on the wall:

$$u = v = 0 \quad \text{on } y = 0. \tag{7}$$

At the interface $y = 1 + \eta$, where η is an interfacial displacement, the tangential stress and normal stress conditions are given by

$$\frac{1}{(1 + \eta_x^2)^{1/2}}[(u_y + v_x)(1 - \eta_x^2) + 2(v_y - u_x)\eta_x^2] = -\mathrm{Ma}\,\Gamma_x + \tau_s, \tag{8}$$

$$-p + \frac{2}{(1 + \eta_x^2)}(v_y + u_x\eta_x^2 - (u_y + v_x)\eta_x) = \frac{1}{\mathrm{Ca}}\frac{\eta_{xx}}{(1 + \eta_x^2)^{3/2}}, \tag{9}$$

where $Ca = \mu U_s^*/\sigma_0^*$ is the capillary number and $\mathrm{Ma} = E/\mathrm{Ca}$ is the Marangoni number. The kinematic condition at the interface is given by

$$v = \eta_t + u\eta_x. \tag{10}$$

For an insoluble surfactant with negligible surface diffusion, the transport equation along the interface is given by[12]

$$\frac{\partial}{\partial t}(\sqrt{1 + \eta_x^2}\,\Gamma) + \frac{\partial}{\partial x}(\sqrt{1 + \eta_x^2}\,\Gamma u) = 0. \tag{11}$$

This is consistent with that derived by Wong et al.[15]

We perturb the velocities, the pressure, the interfacial position, and the surfactant concentration as $u = \bar{u} + u'$, $v = v'$, $p = \bar{p} + p'$, $\eta = \eta'$, and $\Gamma = 1 + \Gamma'$, respectively. We then substitute these quantities into the above governing equations (4)–(6) and boundary conditions (7)–(11), and linearize them with respect to the base states. It is convenient to introduce a perturbed stream function ψ' for the perturbed velocity field such that $u' = \psi'_y$ and $v' = -\psi'_x$. We employ normal modes to investigate the stability of the liquid layer:

$$(\psi', \eta', \Gamma') = (\hat{\phi}(y), \hat{\eta}, \hat{\Gamma})\exp[ik(x - ct)], \tag{12}$$

where k is the wave number of the disturbance and $c = c_r + ic_i$ is the complex wave speed. The imaginary part of c determines the growth rate $s = kc_i$. The system is stable (unstable) when s is positive (negative). The resulting equation

governing the linear stability is the well-known Orr–Sommerfeld equation

$$ik\,\mathrm{Re}[(\bar{u}-c)(D^2-k^2)\hat{\phi}-\hat{\phi}D^2\bar{u}]=(D^2-k^2)^2\hat{\phi}, \qquad (13)$$

where $D\equiv d/dy$. Then, in terms of $\hat{\phi}$, Eqs. (7)–(11) become

$$\hat{\phi}'(0)=\hat{\phi}(0)=0, \qquad (14)$$

$$\hat{\phi}''(1)+k^2\hat{\phi}(1)=-ik\,\mathrm{Ma}\,\hat{\Gamma}-\bar{U}''\hat{\eta}, \qquad (15)$$

$$\hat{\phi}'''(1)-3k^2\hat{\phi}'(1)=\left(2ik\cot\theta_0+\frac{ik^3}{\mathrm{Ca}}+2k^2\bar{U}'\right)\hat{\eta}$$
$$+ik\,\mathrm{Re}[(\bar{U}-c)\hat{\phi}'(1)-\bar{U}'\hat{\phi}(1)], \qquad (16)$$

$$\hat{\phi}(1)=(c-\bar{U})\hat{\eta}, \qquad (17)$$

$$(c-\bar{U})\hat{\Gamma}-\bar{U}'\hat{\eta}-\hat{\phi}'(1)=0. \qquad (18)$$

Here we use the superscript $'$ to represent the y derivatives, and define $\bar{U}\equiv\bar{u}(1)=1+\tau_s$, $\bar{U}'\equiv\bar{u}'(1)=\tau_s$, and $\bar{U}''\equiv\bar{u}''(1)=-2$ for simplicity. Equation (16) is derived from the linearized normal stress condition (9) by eliminating the pressure in terms of the stream function via the linearized x-component momentum equation (5). For each k, the system of (13)–(18) constitutes an eigenvalue problem that can be used to determine the complex wave speed c.

III. LONG-WAVELENGTH STABILITY ANALYSIS

In the limit of long wavelengths ($k\to 0$), we follow the regular perturbation technique first proposed by Yih.[6] The appropriate long wave expansions are

$$\hat{\phi}=\hat{\phi}_0+k\hat{\phi}_1+\dots, \quad \hat{\eta}=\hat{\eta}_0+k\hat{\eta}_1+\dots,$$
$$\hat{\Gamma}=\hat{\Gamma}_0+k\hat{\Gamma}_1+\dots, \quad c=c_0+kc_1+\dots. \qquad (19)$$

We substitute (19) into (13)–(18) and collect the terms in each order of k. At $O(1)$ we have

$$\hat{\phi}_0''''=0, \qquad (20)$$

$$\hat{\phi}_0(0)=\hat{\phi}(0)=0, \qquad (21\mathrm{a})$$

$$\hat{\phi}_0''(1)=-\bar{U}''\hat{\eta}_0, \qquad (21\mathrm{b})$$

$$\hat{\phi}'''(1)=0, \qquad (21\mathrm{c})$$

$$\hat{\phi}_0(1)=(c_0-\bar{U})\hat{\eta}_0, \qquad (21\mathrm{d})$$

$$(c_0-\bar{U})\hat{\Gamma}_0-\bar{U}'\hat{\eta}_0-\hat{\phi}_0'(1)=0. \qquad (21\mathrm{e})$$

The solution to (20) that satisfies (21a)–(21c) is given by

$$\hat{\phi}_0=-\frac{\bar{U}''}{2}y^2\hat{\eta}_0. \qquad (22)$$

Substituting (22) into (21d) and (21e) yields, respectively,

$$\left(c_0-\bar{U}+\frac{1}{2}\bar{U}''\right)\hat{\eta}_0=0, \qquad (23\mathrm{a})$$

$$(c_0-\bar{U})\hat{\Gamma}_0=(\bar{U}'-\bar{U}'')\hat{\eta}_0. \qquad (23\mathrm{b})$$

There are two modes. For $\hat{\eta}_0\neq 0$, c_0 and $\hat{\Gamma}_0$ are given by

$$c_0=2+\bar{U}', \qquad (24\mathrm{a})$$

$$\hat{\Gamma}_0=(2+\bar{U}')\hat{\eta}_0. \qquad (24\mathrm{b})$$

This mode is an "*interface*" mode as in free-falling systems[6–8] since it is triggered by the interfacial deflections in view of the fact that the leading order kinematic condition (23a) determines c_0. The surfactant concentration is in phase (out of phase) with the interface when $2+\bar{U}'>0(<0)$. In addition to the interface mode, there is a "*surfactant*" mode which can be triggered by the surfactant concentration perturbations ($\hat{\Gamma}_0\neq 0$) without necessarily having an interfacial deflection. For this mode, (23a) and (23b) imply

$$c_0=\bar{U}, \qquad (25\mathrm{a})$$

$$\hat{\eta}_0=0. \qquad (25\mathrm{b})$$

Note that c_0 here is determined from the leading order surfactant transport equation (23b). As shown above, the $O(1)$ problem does not contribute to the system's stability because c_0 is real. As we shall see next, the instability is determined by the $O(k)$ problem.

At the $O(k)$ problem, we have the following equations:

$$\hat{\phi}_1''''=i\,\mathrm{Re}[(\bar{u}-c_0)\hat{\phi}_0''-\bar{u}''\hat{\phi}_0], \qquad (26)$$

$$\hat{\phi}_1(0)=\hat{\phi}_1'(0)=0, \qquad (27\mathrm{a})$$

$$\hat{\phi}_1''(1)=-i\mathrm{Ma}\hat{\Gamma}_0-\bar{U}''\hat{\eta}_1, \qquad (27\mathrm{b})$$

$$\hat{\phi}_1'''(1)=2i\cot\theta_0\,\hat{\eta}_0+i\,\mathrm{Re}[(\bar{U}-c_0)\hat{\phi}_0'(1)-\bar{U}'\hat{\phi}_0(1)], \qquad (27\mathrm{c})$$

$$\hat{\phi}_1(1)=(c_0-\bar{U})\hat{\eta}_1+c_1\hat{\eta}_0, \qquad (27\mathrm{d})$$

$$\hat{\phi}_1'(1)=(c_0-\bar{U})\hat{\Gamma}_1-\bar{U}'\hat{\eta}_1+c_1\hat{\Gamma}_0, \qquad (27\mathrm{e})$$

where both Re and Ma are $O(1)$. The solution to (26) that satisfies (27a)–(27c) is given by

$$\hat{\phi}_1=i\,\mathrm{Re}\,\bar{U}''\hat{\eta}_0$$
$$\times\left[-\frac{1}{120}(\bar{U}'-\bar{U}'')y^5+\frac{1}{24}c_0y^4\right]+\frac{a_1}{6}y^3+\frac{a_2}{2}y^2, \qquad (28)$$
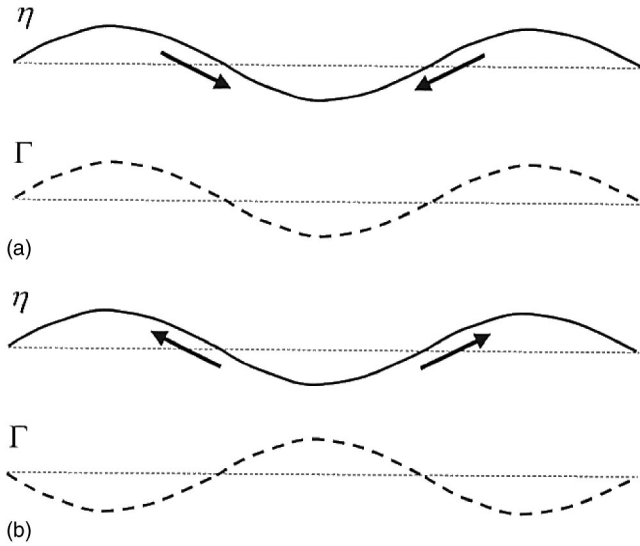
FIG. 2. The mechanisms of stability for the interface mode in view of the phase difference between $\Gamma(=\Gamma_0)$ and $\eta(=\eta_0)$. (a) $2+\bar{U}'>0$, Γ is in phase with η. (b) $2+\bar{U}'<0$, Γ, and η are out of phase. The arrows indicate the directions of resulting Marangoni flows.

$$a_1 = 2i\hat{\eta}_0 \cot\theta_0,$$

$$a_2 = -\bar{U}''\hat{\eta}_1 - i\mathrm{Ma}\hat{\Gamma}_0 + \hat{\eta}_0$$
$$\times\left[-2i\cot\theta_0 + i\,\mathrm{Re}\,\bar{U}''\left(\frac{5}{6}(\bar{U}''-\bar{U}') + \frac{c_0}{2}\right)\right].$$

Below, the first-order wave speed c_1 is determined for both the interface and the surfactant modes.

A. The interface mode

Similar to the $O(1)$ problem, the c_1 for the interface mode is determined from the $O(k)$ kinematic condition (27d). Substituting (28) into (27d), with the aid of (24), yields

$$c_1 = -\frac{2}{3}i\cot\theta_0 + i\frac{4}{15}\mathrm{Re}(2+\bar{U}') - i\frac{1}{2}\mathrm{Ma}(2+\bar{U}'). \quad (29)$$

The first term of (29) represents the stabilizing effect due to the transverse component of gravity. The second term, due to the inertial effect, can be stabilizing if $2+\bar{U}'<0$, that is, when the imposed shear acting against gravity is sufficiently strong. This term is also consistent with the study by Smith.[16] The last term reveals the effect of surfactant, which can be stabilizing as in gravity-driven base flows[7] or destabilizing due to the imposed shears with $\bar{U}'<0$.

Similar to the previous studies,[11–13] the above surfactant effects on the stability can be explained in conjunction with (24b) in view of the phase difference between the interfacial deflection and the surfactant distribution. The mechanism is depicted in Fig. 2. As indicated by (24b), for $2+\bar{U}'>0$ [Fig. 2(a)], $\hat{\Gamma}_0$ has the same sign as $\hat{\eta}_0$; the surfactant concentration perturbation is in phase with the interfacial deflection. That is, the surfactant concentration is higher (lower) at the interface's crest (trough). Such a surfactant concentration generates Marangoni forces pulling the fluid toward the

trough, thereby suppressing the interfacial deflection. Similarly, the out-of-phase configuration for $2+\bar{U}'<0$ [Fig. 2(b)] just acts opposite and thus promotes the interface's growth.

B. The surfactant mode

For the surfactant mode, substituting (25b) into (28) yields $\hat{\phi}_1 = \frac{1}{2}(-i\mathrm{Ma}\hat{\Gamma}_0 - \bar{U}''\hat{\eta}_1)y^2$, and (27d) demands $\hat{\phi}_1(1) = 0$ because of (25a) and (25b), so that

$$i\mathrm{Ma}\hat{\Gamma}_0 = -\bar{U}''\hat{\eta}_1 = 2\hat{\eta}_1, \quad (30a)$$

$$\hat{\phi}_1 = 0. \quad (30b)$$

At $O(k)$, (30a) is a balance between the Marangoni stress $i\mathrm{Ma}\hat{\Gamma}_0$ and the perturbation of base shear stress $-\bar{U}''\hat{\eta}_1$ at the perturbed interface in view of (27b). As a result, for the surfactant mode, since all boundary conditions (27a)–(27d) used for determining the $O(k)$ perturbed flow are zero, these lead to no flow at this order as indicated by (30b). Similar to the $O(1)$ problem, the c_1 for the surfactant mode is determined by the $O(k)$ surfactant transport Eq. (27e). Applying (30a) to (27e) results in

$$c_1 = \frac{i}{2}\mathrm{Ma}\bar{U}'. \quad (31)$$

Unlike the interface mode, the stability is merely determined by whether the imposed shear acts to assist or oppose gravity: the former ($\bar{U}'>0$) destabilizes while the latter ($\bar{U}'<0$) stabilizes.

The above shear-induced Marangoni instability/stability can be interpreted by the $O(k)$ stress balance (30a) in conjunction with the $O(k)$ surfactant transport equation simplified from (27e) using (25a) and (30b):

$$-ik^2 c_1\hat{\Gamma}_0 = -ik^2\bar{U}'\hat{\eta}_1. \quad (32)$$

If we choose the reference frame that moves with the speed $\bar{U}$, and $(\Gamma_0, \eta_1) = (\hat{\Gamma}_0, \hat{\eta}_1)\exp[ik(x-ct)]$, then Eqs. (30a) and (32) are equivalent to $\mathrm{Ma}\Gamma_{0x} = 2k\eta_1$ and $k\Gamma_{0t} = -k\bar{U}'\eta_{1x}$, respectively, in that frame. The schematic mechanism is shown in Fig. 3. For a given sinusoidal interfacial deflection η_1, (30a) suggests that the perturbed interfacial tangential stress of the base flow induces a change in Γ_0 in response to the balance of the Marangoni stress. The resulting surfactant concentration gradient Γ_{0x} is positive (negative) for $\eta_1>0(<0)$. According to (32), Γ_0 is zero at the interface's crest/trough (where $\eta_{1x}=0$) and has maxima/minima at $\eta_1=0$. As a result, Γ_0 has a phase difference of $\pi/2$ with η_1. The system is destabilized if $\bar{U}'>0$ [Fig. 3(a)], since (32) demands that Γ_0 increases (decreases) where $\eta_{1x}<0(>0)$, resulting in an increase in the amplitude of Γ_0. Similarly, the system is stabilized if $\bar{U}'<0$ [Fig. 3(b)].

IV. DISCUSSION AND CONCLUDING REMARKS

We have found two distinct modes that affect the stability of a shear-imposed flow down an inclined plan due to the presence of surfactant. For the interface mode, the Ma-
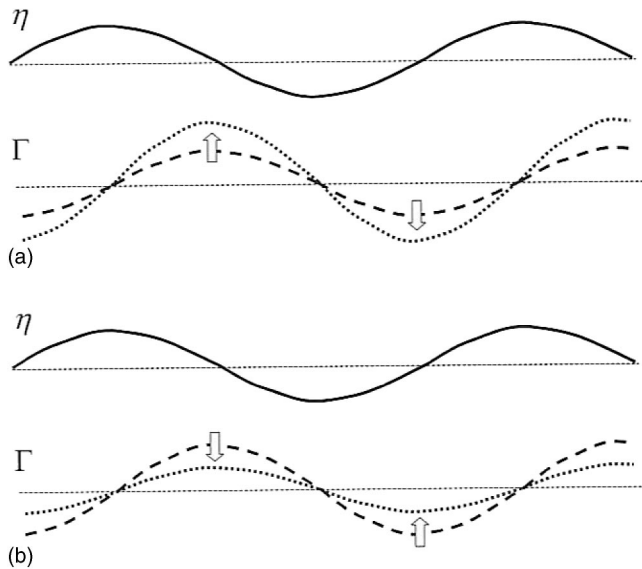
FIG. 3. The mechanism of stability for the surfactant mode. Note that the pictures are drawn in the moving frame with the speed of $\bar{U}=c_0$. $\Gamma(=\Gamma_0)$ has a phase difference $\pi/2$ with $\eta(=k\eta_1)$. (a) $\bar{U}'>0$, (b) $\bar{U}'<0$.

rangoni effect induces a flow that affects the fluid mass balance, leading to the growth or the suppression of the interfacial deflection. For the surfactant mode, however, the Marangoni effect has to incorporate an imposed shear to influence the surfactant mass balance; the stability arises from the accumulation or the reduction of the surfactant concentration in response to the surface convection due to the imposed shear. It is also worth being pointed out that the existence of two modes due to the presence of surfactant has been discussed by Kwak and Pozrikidis.[17]

For the free-falling case $\bar{U}'=0$, only the interface mode survives and the system is stabilized by surfactant for longwaves as in Whitaker and Jones[7] and Lin.[8] Pozrikidis[9] found a less stable system due to surfactant at $O(1)$ wavelengths, suggesting that perhaps this destabilizing effect appears at $O(k^2)$ or higher.

When interfacial shears are introduced for Re=0, inspecting both modes (29) and (31) reveals that for $\bar{U}'<0$ the interface mode is stable whereas the surfactant mode is unstable due to Marangoni effects. This suggests that for a surfactant-laden, free falling film flow that is inherently stable at small Re, imposing a minute interfacial shear along the streamwise direction will cause an instability. Furthermore, in the special case of a strong imposed shear with $|\bar{U}'|\gg 1$ [but still $\gg O(k^{-1})$ for ensuring the validity of the analysis] and Re=0, the interface mode, (29), is $c_1 \approx -i/2\mathrm{Ma}\bar{U}'$, while the surfactant mode, (31), is $c_1 = i/2\mathrm{Ma}\bar{U}'$. The dominant growth rate is thus $s_1 = -ikc_1 = k/2\mathrm{Ma}|\bar{U}'|$ regardless of the direction of the imposed shears, as expected.

Frenkel and Halpern[11] have studied the longwave stability of two-layer Poiseuille–Couette flows in the presence of surfactant. In contrast to our $O(k)$ results, their analysis for the case with a semi-infinite upper layer showed $c=\pm\frac{1}{2}(1$

$-i)|\bar{U}'|^{1/2}\mathrm{Ma}^{1/2}k^{1/2}$ as $k\to 0$. The key of this discrepancy lies in the term $\bar{U}''\eta$ for the perturbed basic shear stress in the interfacial tangential stress condition (8). In their study for two-fluid systems, there is no such term since the basic tangential stress contribution from each fluid cancels out exactly. For a sufficiently strong imposed shear, say, $|\bar{U}'|=O(k^{-1})$, one should expect a similar wave speed as theirs. In this case, the present long wave analysis breaks down and requires a different expansion in the wavenumber k. Indeed, a careful inspection indicates that the leading order wave speed $c\sim\bar{U}\sim O(k^{-1})$ appears to derive from the equation $(c-\bar{U})^2=O(k)$ obtained from the leading order determinant of (17) and (18). The correction to c from the $O(k)$ contribution is thus $O(k^{1/2})$.

In conclusion, we have performed a long wave analysis to examine the effect of an insoluble surfactant on the stability of a shear-imposed falling film flow. Our results reveal that the shear-induced Marangoni effect can destabilize a free falling system that is inherently stable at small Re. Our present study can serve as a useful guide in developing the full stability analysis.

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