# Role of base flows on surfactant-driven interfacial instabilities

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In this paper, we examine how base flows affect interfacial stabilities in the presence of surfactants. A thin-film flow model, subjected to various base-flow conditions, is employed to mimic a wide class of practical interfacial flows. The base flow can be driven by an external force (e.g., pressure forcing or gravity), an interfacial stress, or their combination. For long-wave perturbations, we show that the stability is governed by a coupled set of evolution equations for the interface and surfactant concentration, so the origin of the stability can be unraveled analytically in line with simpler physical arguments. We also demonstrate that the system can exhibit a variety of stability states; it can be neutral, conditionally stable or unstable, or definitely stable or unstable, determined solely by the nature of the base flow and how it regulates surfactant transport. Two modes are found to determine the stability, the interface and surfactant modes, and characterized by the ratio of the basic interfacial shear force to the external force. The routes to the instability are also identified through the action of the base flow. The base flow plays a dual role in affecting the stability: although the imposition of an interfacial shear destabilizes the interface, an external force can cause stabilization. The competition between these two effects gives rise to stability or instability in a range of the force ratio. The underlying mechanisms are elucidated in detail. A generalized criterion for the onset of instability is also established for one- or two-fluid interfacial flows.

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## I. INTRODUCTION

The hydrodynamic stability of coating flows is a subject of long-standing interest in the fundamentals of fluid mechanics as well as in a variety of engineering processes [1]. The issue is often how the interface develops in response to perturbations so as to determine the subsequent fate of the system. In some cases, a liquid can retain its integrity, while in others rupture might occur, depending on flow conditions and fluid properties. It is therefore essential to understand the underlying mechanisms so that one can properly manage instability for a desired process.

In most applications, surfactants or surface-active agents are often present, so their effects can be vital to stability. It is generally believed that the dominant effect of surfactants on stability is the surface-tension-gradient (Marangoni) force due to the nonuniform surfactant distribution along the interface. Since the Marangoni force often serves as a counteracting force to the prevailing flow, the conventional wisdom is that this force tends to recoil the interface from stretch, making the dynamics more sluggish [2–5]. However, as we will demonstrate, this effect is far from trivial under the influences of base flows, which is the main theme of this paper. To see how the problem is motivated, we give a brief account of related research developments below.

Early studies on the stability of surfactant-laden falling film flow have provided some hints to how surfactant affects the stability. A long-wave analysis [2] showed that an insoluble surfactant yields a stabilizing correction to the classical Yih interfacial instability [6]. The stability features can be further complicated by mass transfer processes (e.g., bulk diffusion and sorption) [7,8]. In the case of Stokes flow, a surfactant can make the system less stable than in surfactant-free case [9]. In related work [10], the imposition of "wind shear" is shown to have stabilizing or destabilizing effects when surfactant is present. Studies on two-fluid channel flow with surfactants [11] indicated that a surfactant can destabilize a system that is inherently stable without a surfactant. Such surfactant destabilization is not limited to planar flows, but can also be found in cylindrical flows wherein capillary instability prevails [12]. Indeed, experimental evidence indicates that surfactant-laden systems with applied flows can become more unstable than those without [13]. Similar surfactant-induced instabilities can also be found in oscillating film flows [14] or fingering phenomena [15].

These previous studies suggest that surfactants can have nontrivial effects on stabilities for a wide class of flows. A part of the explanation seems to point toward the response of the Marangoni effect to a base flow. Yet the puzzle here is how a base flow redistributes surfactant along the interface so as to determine the subsequent development of the Marangoni force. Although mechanisms were given in part by some early studies [10–12,16], the detailed route to the instability for given base flow conditions is still not understood. It also entails a rationale to explain the diversity of outcomes found in previous studies. In this work, we consider a surfactant-covered liquid layer flow and examine the stability subjected to a variety of base-flow conditions. As we will show, this simple model suffices to illuminate the role of the base flow in determining the stability characteristics. As we will also demonstrate, the true origin of the stability or instability can be attributed purely to the interplay between the Marangoni effect and base-flow actions.

The paper is organized as follows. We begin with the formulation in Sec. II, and then show stability results for various base-flow conditions in Sec. III. In Sec. IV, we identify how base flows regulate Marangoni effects and mediate the stability. We further generalize our analysis for one- and two-fluid systems in Sec. V. Concluding remarks are made in Sec. VI.

## II. PROBLEM FORMULATION

Consider a liquid layer of density $\rho^*$ and viscosity $\mu^*$ coated on a flat plate. In the unperturbed state, the liquid has a uniform thickness of $h^*$. The air-liquid interface is covered by an insoluble surfactant of a uniform surface concentration $\Gamma_0^*$, and the corresponding surface tension is $\sigma_0^*$. The length, velocity, and pressure (or stress) are scaled by $h^*$, $\sigma_0^*/\mu^*$, and $\sigma_0^*/h^*$, respectively. The system is defined in Cartesian coordinates in which $x$ is the direction aligned with the plane, and $y$ is the outward direction normal to the wall defined at $y=0$. Also let $u$ and $v$ be the velocity components in the $x$ and $y$ directions, respectively, and let $p$ be the pressure. The base flow can be driven by an external force $F$ (e.g., pressure forcing or gravity), by shearing of an interfacial stress $\tau$ (e.g., wind shear or thermocapillary force), or by their combination:

$$U = -\frac{F}{2}(y^2 - 2y) + \tau y. \tag{1}$$

We are interested in five cases: (i) $F>0$ and $\tau=0$, (ii) $F>0$ and $\tau\neq 0$, (iii) $F=0$ and $\tau\neq 0$, (iv) $F=-3\tau/2$, and (v) $Q=F/3+\tau/2>0$ and $\tau<0$. These cases simulate a wide range of practical interfacial flows. Case (i) describes a freely falling film flow [2]. Case (ii) models a falling liquid flow subjected to an additional wind shear. It can also serve as a simplified model to analyze the motion of a liquid layer overlaid by another thicker fluid layer. In this case, part of the thin-layer motion can be thought of as being sheared by the thicker layer. Case (iii) is a pure surface shear flow. Case (iv) simulates a surface shear flow in a closed, shallow cavity in which an adverse pressure gradient must necessarily develop to oppose the applied shear to satisfy the requirement of a zero net flow rate across the layer [17]. Case (v) can model a pump generated by a surface force, e.g., thermocapillarity. The device is sealed at one end but left open at the other. As a surface force is applied toward the closed end, an adverse pressure must be established and hence pump the fluid toward the open end. The required surface force depends on the desired pumping flow rate $Q$.

Now consider the stability problem. Flow quantities are decomposed into the base-state quantities plus small perturbations. By linearizing the system with respect to the base state, the stability problem can be formulated in terms of perturbation quantities. Let $\eta$ and $\Gamma$ be the interfacial and surfactant perturbations, respectively. To facilitate an understanding of the underlying physics, we focus on long-wavelength perturbations, i.e., the wave number $k=2\pi h^*/L^*\ll 1$ with $L^*$ being the wavelength, so the problem can be formulated by lubrication theory. Further, assume that the Bond number $\mathrm{Bo}=\rho^* g h^{*2}/\sigma_0^*$ and the Reynolds number $\mathrm{Re}=\rho^*\sigma_0^* h^*/\mu^{*2}$ are small, so we can exclude buoyancy and inertial effects. This can isolate the problem from these effects and hence allow us to focus on the impact arising merely from the surfactant. We solve the perturbation flow field up to $O(k)$, the lowest order that can capture all ingredients determining the system's stability. This will also reduce the problem to a coupled set of evolution equations for the interface and surfactant concentration. The resulting evolution equations will take form up to $O(k^2)$ and determine the stability at that order.

The governing equations of the perturbation flow are

$$u_x + v_y = 0, \quad u_{yy} = 0, \quad p_y = 0. \tag{2}$$

Here, the capillary pressure due to surface tension is $O(k^2)$ from the normal stress condition at the interface; its effects on the perturbation velocity field are $O(k^3)$ and thus negligible. As a result, $u$ is linear in $y$ and can be determined by the tangential stress condition at the interface:

$$u_y(1) = -U_{yy}(1)\eta - M\Gamma_x, \tag{3}$$

where $M=-(\Gamma_0^*/\sigma_0^*)(\partial\sigma^*/\partial\Gamma^*)_{\Gamma_0^*}$ is the Marangoni number and assumed to be $O(1)$. The resulting perturbation flow field is given by

$$u = (F\eta - M\Gamma_x)y, \quad v = -(F\eta_x - M\Gamma_{xx})y^2/2. \tag{4}$$

We further require the kinematic condition and the surfactant transport equation. In the frame moving with the basic interfacial velocity $U(1)=F/2+\tau$, they are

$$v(1) = \eta_t, \tag{5}$$

$$\Gamma_t + U_y(1)\eta_x + [u(1)]_x = 0. \tag{6}$$

In (6), the surface diffusion is assumed negligible. Applying (4)–(6), we derive a coupled set of evolution equations for the interface and surfactant concentration:

$$\eta_t + \frac{F}{2}\eta_x - \frac{1}{2}M\Gamma_{xx} = 0, \tag{7}$$

$$\Gamma_t + (\tau + F)\eta_x - M\Gamma_{xx} = 0. \tag{8}$$

These equations, though simple, suffice to reveal the essence of the underlying physics and play a central part in this analysis. Three consequences immediately follow. First, a clean-interface flow is neutrally stable, as indicated by Eq. (7) with $\Gamma=0$. Second, a stationary surfactant system ($F=\tau=0$) is stable because of the diffusion nature of Eq. (8). Last, since neither clean-interface flow nor a stationary base state with surfactant gives rise to instability, we conclude that an instability, if any, must arise from effects combining the base flow and surfactant.

## III. STABILITY CHARACTERISTICS UNDER VARIOUS BASE-FLOW CONDITIONS

The stability of Eqs. (7) and (8) is analyzed by taking normal modes $(\eta,\Gamma)=(\hat{\eta},\hat{\Gamma})\exp(ikx+st)$ where $s$ is the complex growth rate whose real part determines the stability. $s$ can be obtained analytically from a quadratic equation. Since the stability or instability is determined by the Marangoni terms that are $O(k^2)$, we expand $s$ up to $O(k^2)$. We now examine the stability for each base-flow condition. For case (i), $F>0$ and $\tau=0$ as in freely falling film, we find $s_1=-ikF/2-k^2M$ and $s_2=0$; thus the system remains neutral because of $s_2$. The first mode is stabilizing and corresponds to the usual Yih mode [2,6]. The second mode arises from the surfactant and impacts the stability at $O(k^3)$.

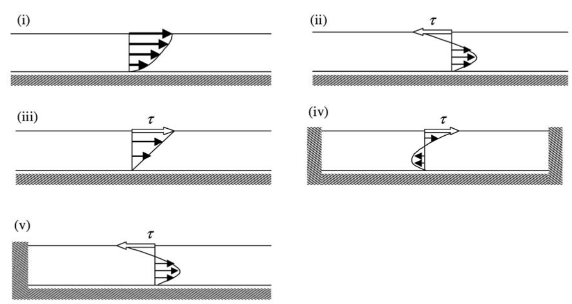
Now consider case (ii), $F>0$ and $\tau\neq 0$. We obtain the growth rates $s_1=-ikF/2-k^2M(1+\tau/F)$ and $s_2=k^2M\tau/F$. Compared to case (i), the role of the second mode in determining the stability is now manifest in the presence of an imposed shear. It is destabilizing (stabilizing) if the direction of the shear is the same as (opposite to) that of the external forcing. For the first mode, however, the effect of the imposed shear is the opposite. Overall, we find an instability if $\tau/F<-1$ or $\tau/F>0$ [note that $\tau/F=-1$ reduces Eq. (8) to a diffusion equation and is clearly stabilizing, and it is also obvious that $\tau/F=0$ recovers case (i)]. The above analysis holds for $\tau/F$ of $O(1)$; it is inapplicable when $\tau/F$ is unbounded, as in case (iii). In case (iii), $F=0$ and $\tau\neq 0$, Eqs. (7) and (8) become $\eta_t-M\Gamma_{xx}/2=0$ and $\Gamma_t+\tau\eta_x-M\Gamma_{xx}=0$, and the corresponding growth rate is $s=\pm(i/2)^{1/2}|\tau|^{1/2}M^{1/2}k^{3/2}$, irrespective of the direction of the applied shear. Hence, the system is completely destabilized by the surfactant. The instability now occurs at $O(k^{3/2})$, which is more dangerous than the $O(k^2)$ found in case (ii). A closer inspection further reveals that the Marangoni term in (8) does not contribute to the leading order growth rate in this case.

As for case (iv), $F=-3\tau/2$, as in shearing of a film within a shallow cavity, we arrive at $s_1=-ikF/2-k^2M/3$ and $s_2=-2k^2M/3$. In contrast to case (iii), which is always unstable due to surfactant, the system now is completely stabilized by surfactant. Although this case can be considered as a special scenario of case (ii) from a mathematical viewpoint, the base flow in part is created by a returning flow due to the zero-net-flow constraint, which differs, in principle, from the unconfined flow case (ii). Finally, we examine case (v), $Q=F/3+\tau/2>0$ and $\tau<0$, for surface-force-driven pumping. It again follows the same growth rates as in case (ii). Yet $\tau/F$ must be less than 0 as the required pressure force for pumping must oppose the applied shear; it thus follows that the system is stable only if $-2/3<\tau/F<0$.

As demonstrated above, a liquid film with surfactant can experience all possible stability states, as summarized in Table I. It can be neutral, conditionally stable or unstable, or definitely stable or unstable, determined solely by the nature of the base flow and by how it regulates the Marangoni effect.

TABLE I. Results of stability with surfactant under various base-flow conditions. These conditions model a wide class of practical flows: (i) freely falling film flow, (ii) thin-film flow sheared by another thicker layer, (iii) pure surface shear flow, (iv) sheared liquid in a shallow cavity, and (v) flow generated by shearing liquid against the wall.

| Case | Growth rates $s_1$ and $s_2$ | Stability state |
|---|---|---|
| (i) $F>0,\tau=0$ | $-ikF/2-k^2M$, 0 | neutral |
| (ii) $F>0,\tau\neq 0$ | $-ikF/2-k^2M(1+\tau/F)$, $k^2M\tau/F$ | stable if $-1<\tau/F<0$ |
| (iii) $F=0,\tau\neq 0$ | $\pm(i/2)^{1/2}\lvert\tau\rvert^{1/2}M^{1/2}k^{3/2}$ | unstable |
| (iv) $F=-3\tau/2$ | $-ikF/2-k^2M/3$, $-2k^2M/3$ | stable |
| (v) $Q=F/3+\tau/2>0$ with $\tau<0$ | same as (ii) | stable if $-2/3<\tau/F<0$ |




## IV. MARANGONI MODULATION INDUCED BY BASE FLOWS

How a base flow mediates the stability of an interface with surfactant is reflected by the $\eta$ terms in Eqs. (7) and (8). As the $\eta$ term of Eq. (7) comes from the jump of the basic interfacial stress $-U_{yy}(1)\eta=F\eta$ in (3), its effect on the fluid mass balance is reflected only by the external force $F$. The $\eta$ term of Eq. (8) is attributed to the surface convection arising from the interface deflection. In addition to the contribution from $F$, it also involves the effect from the perturbation to the basic interfacial velocity, $U_y(1)\eta=\tau\eta$ in (6). Thus the surface advection effect on the surfactant distribution is further mediated by the surface shear $\tau$. As such, the effects of the base flow are twofold. On the one hand, the base flow can rearrange the surfactant distribution [via $(F+\tau)\eta_x$ in (8)], inducing a Marangoni flow to impact the interface dynamics. On the other hand, it can also modulate the interface evolution [via $F\eta_x/2$ in (7)] and hence the subsequent development of the interface. Below, we illustrate how these effects interplay in more detail.

We first discuss the effects of $(F+\tau)\eta_x$ in (8). Consider a system initially subjected to a sinusoidal perturbation of amplitude $\delta\,(\ll 1)$ to the interface. The initial surfactant concentration is assumed uniform. Suppose $F+\tau>0$. The advection of surfactant due to $(F+\tau)\eta_x$ increases (decreases) $\Gamma$ in an amount of $O(k\delta)$ for $\eta_x<0$ ($>0$), making $\Gamma$ lead $\eta$ by $\pi/2$. As a normal Marangoni flow of $O(k^3\delta)$ is induced through $M\Gamma_{xx}/2$ in (7), this flow is out of phase with $\Gamma$ and acts to increase (decrease) $\eta$ on the portion of the interface with $\eta_x>0$ ($<0$). That is, it tends to increase the interface amplitude in such a way that the interface looks as if it is traveling backward. Suppose that the traveling term $F\eta_x/2$ in (7) is absent. Then the amplification in $\eta$ will keep steepening $\Gamma$ while the Marangoni diffusion [of $O(k^3\delta)$] from $M\Gamma_{xx}$ in (8) is too weak to attenuate that steepening [of $O(k\delta)$]. This in turn exaggerates the interface's amplification, and thereby destabilizes the system. As this destabilization persists for a time scale $T$, the interface amplitude will grow like $O(k^3\delta T)$ due to the flow-induced Marangoni effect. Balancing that

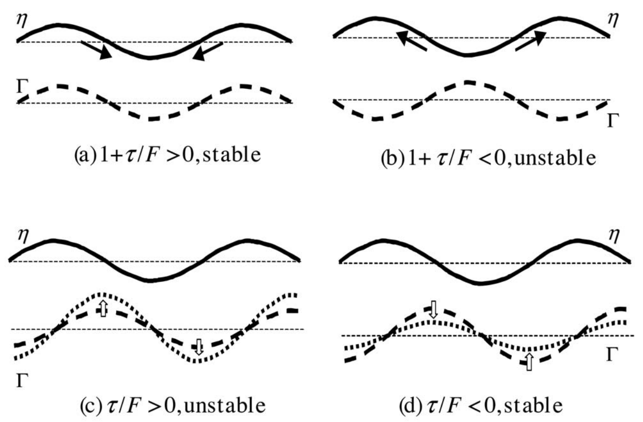



FIG. 1. Mechanisms of base-flow regulation of the Marangoni instability. Interface mode (a), (b) and surfactant mode (c), (d). Arrows along the interface $\eta$ show the directions of Marangoni forces; those in $\Gamma$ indicate changes in the surface concentration of surfactant.

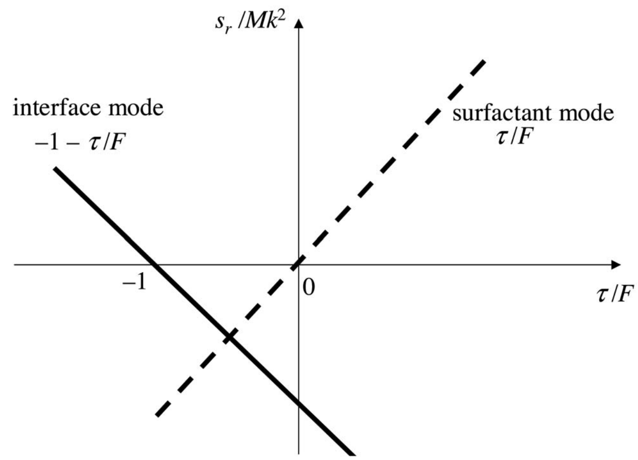



FIG. 2. Long-wave growth rates for the interface and surfactant modes. The competition between the two modes leads to a stability window $-1<\tau/F<0$.

growth with $\eta_t \sim O(\delta T^{-1})$ in (7), we find $T\sim O(k^{-3/2})$ in accord with the $O(k^{3/2})$ growth rate found in case (iii).

As explained above, the base flow can rearrange the surfactant distribution through $(F+\tau)\eta_x$ in (8), and the induced Marangoni effect tends to stimulate interface growth, as manifested by the absence of $F\eta_x/2$ in (7). If $F\eta_x/2$ comes into play, it will make the interface travel at $O(k)$, hence overriding the aforementioned base-flow-induced Marangoni promotion of the interface growth. At $O(k)$, viz., neglecting Marangoni effects, Eqs. (7) and (8) suggest $\Gamma_t=2(1+\tau/F)\eta_t$. As a result, there is no instability and $\Gamma$ simply travels with $\eta$ in one of two ways: either in phase with $\eta$ for $1+\tau/F>0$, or out of phase with $\eta$ for $1+\tau/F<0$. As the stability lies in the $O(k^2)$ Marangoni terms, the $\Gamma$-$\eta$ phase configuration at $O(k)$ will determine the way Marangoni forces act on the interface [see Figs. 1(a) and 1(b)]. If $\Gamma$ and $\eta$ are in phase, the Marangoni forces tend to pull the fluid away from the peaks of $\eta$, which decreases the $\eta$ amplitude and hence is stabilizing. On the other hand, the out-of-phase case will be destabilizing because the corresponding Marangoni action is the opposite. Since this explains the stability corresponding to the $s_1$ mode due to the traveling interface, we refer to it as an *interface* mode.

In addition to the interface mode above, Eqs. (7) and (8) also admit a stationary solution (i.e., $s=0$) at $O(k^0)$ even when $F\neq 0$. At $O(k^0)$, the interface is flat, but permits a nonzero surfactant perturbation $\Gamma$. In contrast to the interface mode, the base flow does not participate in the $O(k^0)$ solution; its effects come into the $O(k^2)$ problem. At $O(k^2)$, Eq. (8) suggests that the $O(k^0)$ $\Gamma$ sets off a Marangoni effect of $O(k^2)$ to induce an interfacial deflection $\eta$ of $O(k)$ through the base flow. This yields $F\eta_x - M\Gamma_{xx}=0$ [provided $F$ is $O(1)$] leading $\eta$ to lag behind $\Gamma$ by $\pi/2$. From (8), the base flow's advection set by this interfacial deflection redistributes $\Gamma$ along the interface, causing $\Gamma$ to be in or out of phase with $\eta_x$. Combining Eqs. (7) and (8) we have $\Gamma_t+(\tau/F)M\Gamma_{xx}=0$ which is just the $s_2$ mode. It is simply a diffusion equation that admits stability for $\tau/F<0$ and instability otherwise [see Figs. 1(c) and 1(d)]. Note that the influence of the base flow on the stability only appears as $\tau/F$, as a result of the interface movement $F\eta_x$ in (7) followed by the advection of surfactant $(F+\tau)\eta_x$ in (8). Since the stability here arises from the imbalance of the surfactant mass, we call this mode the *surfactant* mode. A similar mechanism can also serve as an alternative account for the $F\to 0$ limit (i.e., large $|\tau|/F$) discussed earlier, but Eqs. (7) and (8) take a form in which $M\Gamma_{xx}$ is neglected in (8) and $F=0$.

As shown above, the base flow influences the stability of an interface with a surfactant through two modes, the interface and surfactant modes. In the interface mode, a traveling interface accommodates a surfactant wave that develops Marangoni forces to affect the subsequent fate of the interface. The stability depends on the phase difference between these two waves, and is modulated through the base flow by $1+\tau/F$: $1+\tau/F>0$ ($<0$) stabilizes (destabilizes). In the surfactant mode, a surfactant perturbation triggers an interfacial deflection. As the surfactant distribution is rearranged by the base flow's advection, the imbalance of the surfactant mass will determine the stability. The stability depends on whether $\tau$ acts to assist or oppose $F$: $\tau/F>0$ ($<0$) destabilizes (stabilizes). As such, both modes are characterized only by $\tau/F$, but the corresponding effects on the stability differ, as shown in Fig. 2. As increasing the magnitude of $\tau/F$ enhances the effect for one mode but does the opposite for the other, the competition will lead to instability if $\tau/F<-1$ (the interface mode dominates) or $\tau/F>0$ (the surfactant mode dominates). Therefore, imposing an interfacial shear—even if it is minuscule—in favor of the flow driven by an external forcing will activate an instability. However, if the surface shear acts to oppose the external forcing, the system can be stabilized unless the former is sufficiently strong. In view of the above, we conclude that, when surfactant is present, the effect of $\tau$ tends to induce destabilization, but $F$ renders stabilization since increasing its magnitude discourages the instability. Thus, the reason why a surfactant-laden interfacial flow can exhibit a diversity of stability states under different base-flow conditions can now be understood. In case (i), there is no interfacial shear; hence no instability can be activated. Case (ii) is a general scenario, as discussed above. In case (iii), the shear-induced Marangoni destabilization prevails because there is no external forcing to mitigate the instability. Case (iv) is stable since the developed adverse pressure gradient is sufficiently strong to dampen the instability excited by the interfacial shear. Case (v) is conditionally stable in a range of $\tau/F$, as $F$ must necessarily oppose $\tau$ and a too large $\tau$ will destabilize the interface.

## V. GENERALIZED VIEW OF STABILITY IN ONE- AND TWO-FLUID INTERFACIAL FLOW WITH A SURFACTANT

Although the Marangoni modulation of interfacial stability was illustrated on the basis of a single-fluid flow, similar effects could also, in principle, appear in two-fluid flows. In that case, a similar set of evolution equations can be derived by further taking into account the fluid volumes and differences between the fluids' mechanical properties [16]. In fact, for most single- or two-fluid flows, we find that the evolution equations that govern the long-wave stability with surfactant can be written in the generalized form

$$\eta_t + \alpha \eta_x - \beta M \Gamma_{xx} = 0, \tag{9}$$

$$\Gamma_t + \gamma \eta_x - \nu M \Gamma_{xx} = 0. \tag{10}$$

These equations also appear in cylindrical systems at sufficiently low surface tension [12]. Again, the effects of the base flow are reflected by $\alpha$ and $\gamma$. Note that $\nu$ must be positive to assure Marangoni-diffusion damping in the absence of base flows. Using normal mode analysis, we find that an instability will occur if the following criterion is satisfied

$$\beta\gamma(\beta\gamma - \alpha\nu) > 0. \tag{11}$$

It is worth noting that (11) does not depend on $M$ even though the instability requires the participation of Marangoni effects. As suggested by (11), the base-flow-induced Marangoni effect $\beta\gamma$ is necessary to instability; but it is mediated by $\alpha\nu$—the combined effects of the traveling interface wave and the Marangoni diffusion. Thus, the generalization of the Marangoni modulation on the stability is now furnished.

For our model equations (7) and (8), applying $(\alpha,\beta,\gamma,\nu)=(F/2,1/2,(F+\tau)/2,1)$ to the criterion (11), we find $(F+\tau)\tau > 0$ for the onset of instability. This suggests that the base flow has two competing effects on the stability: while an interfacial shear $\tau$ tends to destabilize because $\tau^2 > 0$, an external force $F$ can be stabilizing if $F$ acts to oppose $\tau$. It is this competition responsible for the variety of stability states.

## VI. CONCLUDING REMARKS

We have studied the interfacial stability of a surfactant-laden liquid film flow. The emphasis is placed on understanding the Marangoni instability due to the nontrivial interplay between surfactant and base flows. Using a long-wave theory, we reduce the stability problem to a coupled set of evolution equations for the interface and surfactant concentration perturbations. The stability characteristics are demonstrated under a variety of base-flow conditions that can simulate a wide class of practical interfacial flows. The results reveal that the system can experience all possible stability states, determined solely by the nature of the base flow and how it regulates the Marangoni actions. More importantly, the instability can occur in the absence of buoyancy (i.e., Rayleigh-Taylor instability) and inertial effects. For cylindrical interfacial flows where capillary instability often dominates, this flow-induced Marangoni destabilization can make the flows even more unstable [12]. There is also an implication for coating flows. As the interface of a thin film could be surfactant rich or susceptible to surface contamination, the stability or instability arising from different flow conditions will determine whether the film ruptures or retains its integrity, which is critical to the efficiency of a coating process.

We show that the stability characteristics are determined by the interface and surfactant modes. As the behaviors of these modes can be characterized solely by the force ratio of the imposed interfacial shear to the external forcing, we find that the base flow plays a dual role in affecting the stability of an interface with a surfactant. On the one hand, the imposition of a surface shear tends to induce destabilization; on the other hand, an external force can cause stabilization. Therefore, the competition between the two modes gives rise to a variety of stability states in a range of the force ratio.

Although previous studies on one- or two-fluid flows have revealed some features of the flow-induced Marangoni instability [2,7–16], our one-fluid ansatz gives a simple but comprehensive account of the effects at work. It not only elucidates how base flows mediate stability of an interface with a surfactant, but also provides a rationale to explain the diversity of stability results under various flow conditions.

## ACKNOWLEDGMENT

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